Pressure Vessel Design Calculations, DRAFT Stainless steel vessel with Copper Liner D. Shuman, Feb 21, 2012

Active volume dimensions, from earlier analyses:

$$r_{Xe} = 0.53 \,\text{m}$$
 $l_{Xe} = 1.3 \,\text{m}$

We consider using a field cage solid insulator/light tube of 3 cm total thk., and a copper liner of 12 cm thickness, including all tolerances and necessary gaps.

$$t_{fc} := 3 \text{cm}$$
 $t_{Cu} := 12 \text{cm}$

Pressure Vessel Inner Radius:

$$R_{i_pv} := r_{Xe} + t_{fc} + t_{Cu}$$
 $R_{i_pv} = 0.68 \text{ m}$

Temperatures:

For pressure operation, the temperature range will be 10C-30C. For vacuum operation, the temperature range will be 10C to 150C (bakeout).

Vessel wall thicknesses

Material:

We use 316Ti for vessel shells and flanges due to its known good radiopurity and strength.

Design Rules: division 1

316Ti is not an allowed material under section VIII, division 2, so we must use **division 1** rules. The saddle supports are however, deisgned using the methodilogy given in div. 2, as div. 1 does not provide design formulas (nonmandatory Appendix G)

Maximum allowable material stress, for sec. VIII, division 1 rules from ASME 2009 Pressure Vessel code, sec. II part D, table 1A:

Youngs modulus

$$S_{max_316Ti_div1} := 20000psi - 20F - 100F$$
 $E_{SS_aus} := 193GPa$

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input check result (all conditions should be true (=1)

$$xx := 1 \quad xx > 0 = 1$$

Choose material:

$$S_{max} := S_{max_316Ti_div1}$$

Maximum Operating Pressure (MOP), gauge:

$$MOP_{pv} := (P_{MOPa} - 1bar)$$
 $MOP_{pv} = 14 bar$

Minimum Pressure, gauge:

 $P_{min} = -1.5 \, bar$ the extra 0.5 atm maintains an upgrade path to a water or scintillator tank

Maximum allowable pressure, gauge (from LBNL Pressure Safety Manual, PUB3000) at a minimum, 15% over max operating pressure; this is design pressure at LBNL:

$$MAWP_{DV} := 1.1MOP_{DV}$$
 $MAWP_{DV} = 15.4 \text{ bar}$

Vessel wall thickness, for internal pressure is then (div 1), Assume all welds are type (1) as defined in UW-12, are fully radiographed so weld efficiency:

$$E_W := 1$$
 <---note: it is not clear if butt welds using ceramic backing strips will qualify as type (1)

Minimum wall thickness is then:

$$t_{pv_d1_min_ip} := \frac{MAWP_{pv} \cdot \left(R_{i_pv}\right)}{S_{max} \cdot E_w - 0.6 \cdot MAWP_{pv}}$$

$$t_{pv_d1_min_ip} = 7.75 \text{ mm}$$

$$t_{pv d1 min ip} = 7.75 mm$$

We set wall thickness to be:

$$t_{pv} := 8mm$$
 t_{pv}

$t_{pv} > t_{pv_d1_min_ip} = 1$

Maximum Allowable External Pressure

ASME PV code Sec. VIII, Div. 1- UG-28 Thickness of Shells under External Pressure

Maximum length between flanges

$$L_{ff} := 1.6m$$

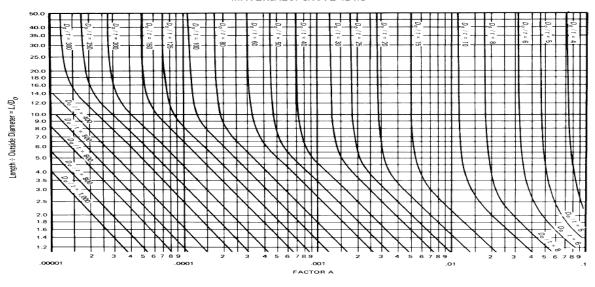
The maximum allowable working external pressure is determined by the following procedure:

Compute the following two dimensionless constants:

$$\frac{L_{ff}}{2R_{i_pv}} = 1.2 \qquad \frac{2R_{i_pv}}{t_{pv}} = 170$$

From the above two quantities, we find, from fig. G in subpart 3 of Section II, the factor A:

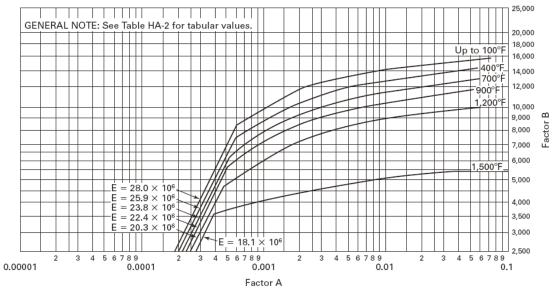
FIG. G GEOMETRIC CHART FOR COMPONENTS UNDER EXTERNAL OR COMPRESSIVE LOADINGS (FOR ALL MATERIALS) [NOTE (14)]



A := 0.0005

Using the factor A in chart (HA-2) in Subpart 3 of Section II, Part D, we find the factor B (@ 400F, since we may bake while pulling vacuum):

FIG. HA-2 CHART FOR DETERMINING SHELL THICKNESS OF COMPONENTS UNDER EXTERNAL PRESSURE DEVELOPED FOR AUSTENITIC STEEL 16Cr-12Ni-2Mo, TYPE 316



B := 6200psi

@ 400F

The maximum allowable working external pressure is then given by :

$$P_a := \frac{4B}{3\left(\frac{2R_{i_pv}}{t_{pv}}\right)}$$

 $P_a = 3.3 \,\text{bar}$ $-P_{min} = 1.5 \,\text{bar}$

$$P_a > -P_{min} = 1$$

Flange thickness:

inner radius max. allowable pressure

 $R_{i~pv} = 0.68 \, m$ MAWP_{pv} = 15.4 bar (gauge pressure)

The flange design for helicoflex or O-ring sealing is "flat-faced", with "metal to metal contact outside the bolt circle". This design avoids the high flange bending stresses found in a raised face flange (of Appendix 2) and will result in less flange thickness, even though the rules for this design are found only in sec VIII division 1 under Appendix Y, and must be used with the lower allowable stresses of division 1.

Flanges and shells will be fabricated from 316Ti, 304L or 316L (ASME spec SA-240) stainless steel plate. Plate samples will be helium leak checked before fabrication, as well as ultrasound inspected. The flange bolts and nuts will be inconel 718, (UNS N77180) as this is the highest strength non-corrosive material allowed for bolting.

We will design to use one Helicoflex 5mm gasket (smallest size possible) with aluminum facing (softest) loaded to the minimum force required to achieve helium leak rate.

Maximum allowable material stresses, for sec VIII, division 1 rules from ASME 2010 Pressure Vessel code, sec. II part D, table 2B:

Maximum allowable design stress for flange

$$S_f := S_{max 316Ti div1}$$
 $S_f = 137.9 MPa$

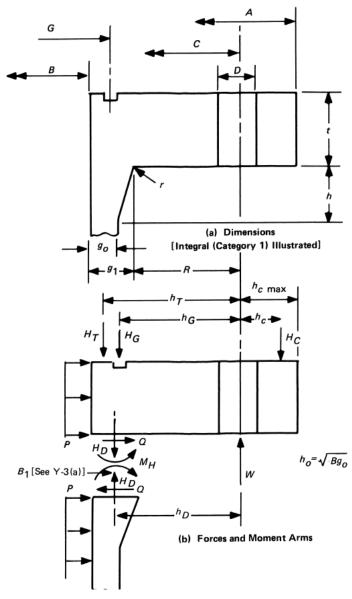
Maximum allowable design stress for bolts, from ASME 2010 Pressure Vessel code, sec. II part D, table 3

Inconel 718 (UNS N07718) $S_{\text{max}}N07718 := 37000 \text{psi}$

$$S_b := S_{max N07718}$$
 $S_b = 255.1 \text{ MPa}$

From sec. VIII div 1, non-mandatory appendix Y for bolted joints having metal-to-metal contact outside of bolt circle. First define, per Y-3:

FIG. Y-3.2 FLANGE DIMENSIONS AND FORCES



hub thickness at flange (no hub)

corner radius:

$$g_0 := t_{pv} \quad g_1 := t_{pv} \quad g_0 = 8 \text{ mm}$$

$$g_1 = 8 \text{ mm}$$

$$\mathbf{g}_0 \coloneqq \mathbf{t}_{pv} \quad \mathbf{g}_1 \coloneqq \mathbf{t}_{pv} \quad \mathbf{g}_0 = 8 \text{ mm} \qquad \mathbf{g}_1 = 8 \text{ mm} \qquad \mathbf{r}_1 \coloneqq \max \left(.25 \mathbf{g}_1, 5 \text{mm}\right) \mathbf{r}_1 = 5 \text{ mm}$$

Flange OD

$$A := 1.47m$$

Flange ID

$$B := 2R_{i_pv}$$
 $B = 1.36 m$

$$B_1 := B + g_1$$
 $B_1 = 1.368 \,\mathrm{m}$

Bolt circle (B.C.) dia, C:

$$C := 1.43 \cdot m$$

Gasket dia

$$G := 2(R_{i pv} + .75cm)$$
 $G = 1.375 m$

Force of Pressure on head

$$H := .785G^2 \cdot MAWP_{pv}$$
 $H = 2.316 \times 10^6 N$

Sealing force, per unit length of circumference:

for O-ring, 0.275" dia., shore A 70 F= ~5 lbs/in for 20% compression, (Parker O-ring handbook); add 50% for smaller second O-ring. (Helicoflex gasket requires high compression, may damage soft Ti surfaces, may move under pressure unless tightly backed, not recommended)

Helicoflex has equivalent formulas using Y as the unit force term and gives several possible values.

for 5mm HN200 with aluminum jacket:

$$Y_1 := 70 \frac{N}{mm} \text{ min value for our pressure and required leak rate (He)} \qquad Y_2 := 220 \frac{N}{mm} \quad \text{recommended value for large diameter seals, regardless of pressure or leak rate}$$

for gasket diameter $D_i := G$ $D_i = 1.375 \, m$

Force is then either of:

$$F_m := 2\pi D_j \cdot Y_1$$
 or $F_j := 2\pi \cdot D_j \cdot Y_2$
 $F_m = 6.048 \times 10^5 \text{ N}$ $F_j = 1.901 \times 10^6 \text{ N}$

Helicoflex recommends using Y2 (220 N/mm) for large diameter seals, even though for small diameter one can use the greater of Y1 or Ym=(Y2*(P/Pu)). For 15 bar Y1 is greater than Ym but far smaller than Y2. Sealing is less assured, but will be used in elastic range and so may be reusable. Flange thickness and bolt load increase quite substantially when using Y2 as design basis, which is a large penalty. We plan to recover any Xe leakage, as we have a second O-ring outside the first and a sniff port in between, so we thus design for Y1 (use F_m) and "cross our fingers" : if it doesn't seal we use an O-ring instead and recover permeated Xe with a cold trap. Note: in the cold trap one will get water and N2, O2, that permeates through the outer O-ring as well.

Start by making trial assumption for number of bolts, root dia., pitch, bolt hole dia D,

$$n := 140$$
 $d_b := 14.77 mm$

Choosing ISO fine thread, with pitch; thread depth:

$$p_t := 1.0 \text{mm}$$
 $d_t := .614 \cdot p_t$

Nominal bolt dia is then;

$$d_{b_nom_min} := d_b + 2d_t$$
 $d_{b_nom_min} = 15.998 \,\text{mm}$

Set:

$$d_{b_nom} := 16mm$$
 $d_{b_nom} > d_{b_nom_min} = 1$

Check bolt to bolt clearance, here we use narrow thick washers under nuts with for box wrench b2b spacing is 1.2 in for 1/2in bolt twice bolt dia (2.4xd_b):

$$\pi C - 1.95 \text{n} \cdot d_{\text{b, nom}} \ge 0 = 1$$

Check nut, washer clearance: $OD_w := 2d_{b-nom}$ this covers the nut width across corners

$$0.5C - (0.5B + g_1 + r_1) \ge 0.5OD_w = 1$$

Flange hole diameter, minimum for clearance:

$$D_{tmin} := d_{b nom} + 2mm$$
 $D_{tmin} = 18 mm$

$$D_t := 19.4 \text{mm}$$

$$D_t > D_{tmin} = 1$$

note: actual clearance holes will be 18mm but some holes will be tapped for M20x1 instead (19.4mm avg dia), so as to allow bolting up the head retraction fixture. These holes should be sleeved when not in use to avoid thread interference with flange bolts.

Compute Forces on flange:

We use a unit gasket seating force of Y1 above

$$\begin{split} & H_{G} \coloneqq F_{m} & H_{G} = 6.048 \times 10^{5} \, \mathrm{N} \\ & h_{G} \coloneqq 0.5 \big(\mathrm{C} - \mathrm{G} \big) & h_{G} = 2.75 \, \mathrm{cm} \\ & H_{D} \coloneqq .785 \cdot \mathrm{B}^{2} \cdot \mathrm{MAWP_{pv}} & H_{D} = 2.266 \times 10^{6} \, \mathrm{N} \\ & h_{D} \coloneqq \mathrm{D_{t}} & h_{D} = 1.94 \, \mathrm{cm} \\ & H_{T} \coloneqq \mathrm{H} - \mathrm{H_{D}} & H_{T} = 5.027 \times 10^{4} \, \mathrm{N} \\ & h_{T} \coloneqq 0.5 \big(\mathrm{C} - \mathrm{B} \big) & h_{T} = 35 \, \mathrm{mm} \end{split}$$

Total Moment on Flange

$$M_P := H_D \cdot h_D + H_T \cdot h_T + H_G \cdot h_G$$
 $M_P = 6.236 \times 10^4 J$

Appendix Y Calc

$$P := MAWP_{pv}$$
 $P = 1.561 \times 10^6 Pa$

Choose values for plate thickness and bolt hole dia:

$$t := 3.9 \text{cm}$$
 $D := D_t$ $D = 1.94 \text{ cm}$

Going back to main analysis, compute the following quantities:

$$\begin{split} \beta &\coloneqq \frac{C + B_1}{2B_1} \qquad \beta = 1.023 \quad h_C \coloneqq 0.5 \big(A - C \big) \qquad h_C = 0.02 \, m \\ a &\coloneqq \frac{A + C}{2B_1} \quad a = 1.06 \qquad AR \coloneqq \frac{n \cdot D}{\pi \cdot C} \quad AR = 0.605 \qquad h_0 \coloneqq \sqrt{B \cdot g_0} \\ r_B &\coloneqq \frac{1}{n} \bigg(\frac{4}{\sqrt{1 - AR^2}} \, a tan \bigg(\sqrt{\frac{1 + AR}{1 - AR}} \bigg) - \pi - 2AR \bigg) \qquad r_B = 8.738 \times 10^{-3} \\ h_0 &= 0.104 \, m \end{split}$$

We need factors F and G, most easily found in figs 2-7.2 and 7.3 (Appendix 2)

since
$$\frac{g_1}{g_0} = 1$$
 these values converge to $F := 0.90892 \text{ V} := 0.550103$

Y-5 Classification and Categorization

We have identical (class 1 assembly) integral (category 1) flanges, so from table Y-6.1, our applicable equations are (5a), (7)-(13),(14a),(15a),16a)

$$J_{S} := \frac{1}{B_{1}} \left(\frac{2 \cdot h_{D}}{\beta} + \frac{h_{C}}{a} \right) + \pi r_{B} \qquad J_{S} = 0.069 \qquad J_{P} := \frac{1}{B_{1}} \left(\frac{h_{D}}{\beta} + \frac{h_{C}}{a} \right) + \pi \cdot r_{B} \qquad J_{P} = 0.055$$

$$F:=\frac{g_0^2(h_0+F\cdot t)}{V} \qquad F'=1.626\times 10^{-5}\,\text{m}^3 \qquad M_P=6.236\times 10^4\,\text{N}\cdot \text{m}$$

$$A=1.47\,\text{m} \qquad B=1.36\,\text{m}$$

$$K:=\frac{A}{B} \qquad K=1.081 \quad Z:=\frac{K^2+1}{K^2-1}\,Z=12.883$$

$$f:=1$$

$$t_s:=0\text{mm} \quad \text{no spacer}$$

$$1:=2t+t_s+0.5d_b \qquad 1=8.539\,\text{cm} \quad A_b:=n\cdot.785d_b^2$$

$$\text{http://www.hightempmetals.com/tech-sec Y-6.2(a)(3)} \qquad \text{Elastic constants} \qquad \text{data/hitemplnconel718data.php}$$

$$E:=E_{SS_aus} \qquad E_{Inconel_718}:=208\,\text{GPa} \qquad E_{bolt}:=E_{Inconel_718}$$

$$(7-13) \qquad M_S:=\frac{-J_P\cdot F\cdot M_P}{2} \qquad M_S=-924.5\,\text{J}$$

$$M_{S} := \frac{-J_{P} \cdot F' \cdot M_{P}}{t^{3} + J_{S} \cdot F'}$$

$$M_{S} = -924.5 J$$

$$\theta_{B} := \frac{5.46}{E \cdot \pi t^{3}} (J_{S} \cdot M_{S} + J_{P} \cdot M_{P}) \quad \theta_{B} = 5.12 \times 10^{-4}$$

$$E \cdot \theta_{B} = 98.816 \text{ MPa}$$

$$H_{C} := \frac{M_{P} + M_{S}}{h_{C}} \qquad H_{C} = 3.072 \times 10^{6} \text{ N}$$

$$W_{m1} := H + H_G + H_C$$
 $W_{m1} = 5.993 \times 10^6 \,\mathrm{N}$

Compute Flange and Bolt Stresses

$$\begin{split} \sigma_b &\coloneqq \frac{W_{m1}}{A_b} \qquad \sigma_b = 250 \, \text{MPa} \qquad S_b = 255.1 \, \text{MPa} \\ r_E &\coloneqq \frac{E}{E_{bolt}} \qquad r_E = 0.928 \\ S_i &\coloneqq \sigma_b - \frac{1.159 \cdot h_C^2 \cdot \left(M_P + M_S\right)}{a \cdot t^3 \cdot r_E \cdot B_1} \qquad S_i = 245.8 \, \text{MPa} \\ S_{R_BC} &\coloneqq \frac{6 \left(M_P + M_S\right)}{t^2 \left(\pi \cdot C - n \cdot D\right)} \qquad S_{R_BC} = 136.4 \, \text{MPa} \\ S_{R_ID1} &\coloneqq -\left(\frac{2F \cdot t}{h_0 + F \cdot t} + 6\right) \cdot \frac{M_S}{\pi B_1 \cdot t^2} \qquad S_{R_ID1} = 0.92 \, \text{MPa} \end{split}$$

$$S_{T1} := \frac{t \cdot E \cdot \theta_B}{B_1} + \left(\frac{2F \cdot t \cdot Z}{h_0 + F \cdot t} - 1.8\right) \cdot \frac{M_S}{\pi B_1 \cdot t^2} \qquad S_{T1} = 2.15 \text{ MPa}$$

$$s_{T3} := \frac{t \cdot E \cdot \theta_B}{B_1} \qquad s_{T3} = 2.817 \text{MPa}$$

$$S_{\mathbf{H}} := \frac{h_0 \cdot \mathbf{E} \cdot \boldsymbol{\theta}_{\mathbf{B}} \cdot \mathbf{f}}{0.91 \left(\frac{g_1}{g_0}\right)^2 B_1 \cdot \mathbf{V}}$$

$$S_{\mathbf{H}} = 15.051 \, \text{MPa}$$

Y-7 Flange stress allowables: $S_f = 137.9 \,\mathrm{MPa}$

(a)
$$\sigma_b < S_b = 1$$

(2) not applicable

(c)
$$S_{R_BC} < S_f = 1$$

$$S_{R_ID1} < S_f = 1$$

$$\begin{array}{ll} \text{(d)} & S_{T1} < S_f = 1 \\ \\ S_{T3} < S_f = 1 \end{array}$$

(e)
$$\frac{S_{H} + S_{R_BC}}{2} < S_{f} = 1$$

$$\frac{S_{H} + S_{R_ID1}}{2} < S_{f} = 1$$

(f) not applicable

Bolt force

$$F_{bolt} := \sigma_b \cdot .785 \cdot d_b^2$$
 $F_{bolt} = 9.623 \times 10^3 \, lbf$

Bolt torque required, minimum:

$$T_{bolt_min} = 0.2F_{bolt} \cdot d_b$$
 $T_{bolt_min} = 126.4 \, \text{N} \cdot \text{m}$ $T_{bolt_min} = 93.3 \, \text{lbf} \cdot \text{ft}$ for pressure test use 1.5x this value

This is the minimum amount of bolt preload needed to assure joint does not open under pressure. and additional amount for bolt preload is needed to maintain a minimum shear resistance to assure head does not slide downward from weight. Non-mandatory Appendix S of div. 1 makes permissible higher bolt stresses than indicated above when needed to assure full gasket sealing and other conditions. This is consistent with proper preloaded joint practice, in which, for properly designed joints where connction stiffness is much grater than bolt xstiffness,

Additional Calculations:

Shear stress in inner flange lip from shield (could happen only if flange bolts come loose, joint opens under presure or are left loose, otherwise friction of faces will support shield)

masses of Copper shielding in cyl and heads

$$\begin{split} t_{Cu} &= 0.12 \, m & \rho_{Cu} = 9 \times 10^3 \, \frac{kg}{m^3} \\ M_{sh_head} &\coloneqq \rho_{Cu} \cdot \pi R_{i_pv}^2 \cdot t_{Cu} & M_{sh_head} = 1.569 \times 10^3 \, kg \\ M_{sh_cyl} &\coloneqq \rho_{Cu} \cdot 2\pi \cdot R_{i_pv} \cdot t_{Cu} \cdot L_{ff} & M_{sh_cyl} = 7.383 \times 10^3 \, kg \\ t_{lip} &\coloneqq 3 mm \end{split}$$

Shear stress in lip:

$$\tau_{lip} := \frac{M_{sh_head} \cdot g}{R_{i_pv} \cdot t_{lip}} \qquad \qquad \tau_{lip} = 7.542 \, \text{MPa}$$

Shear stress on O-ring land (section between inner and outer O-ring), from pressurized O-ring. This is assumed to be the primary stress. There is some edge moment but the "beam" is a very short one. This shear stress is not in the same direction as the nominal tangential (hoop) stress of the flange.

$$\begin{split} t_{land_radial} &:= .36\text{cm} & w_{land_axial} := .41\text{cm} \\ F_{O_ring_land} &:= 2\pi R_{i_pv} \cdot w_{land_axial} \cdot P \\ A_{O_ring_land} &:= 2\pi R_{i_pv} \cdot t_{land_radial} \\ \tau_{land} &:= \frac{F_{O_ring_land}}{A_{O_ring_land}} & \tau_{land} = 1.778\,\text{MPa} \end{split}$$

Support Design using rules of div 2, part 4.15:

From the diagram below the rules are only applicable to flange attached heads if there is a flat cover or tubesheet inside, effectively maintaining the flanges circular. Since the PMT carrier plate and shielding is firmly bolted in, it serves this purpose and we may proceed. We must also compute the case with the heads attached, as there will be additional load

a) Design Method- although not specifically stated, the formulas for bending moments at the center and at the supports are likely based on a uniform loading of the vessel wall from the vessel contents. In this design, the internal weight (primarily of the copper shield) is applied at the flanges; there is no contact with the vessel shell. We calculate both ways and take the worst case.

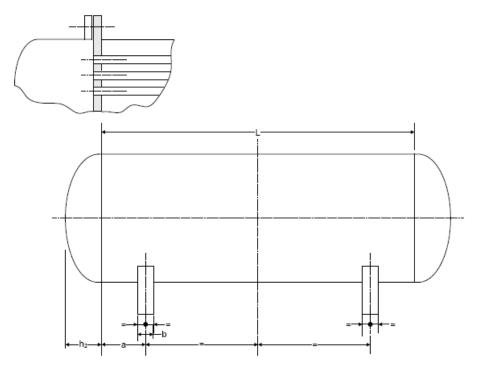


Figure 4.15.1 - Horizontal Vessel on Saddle Supports

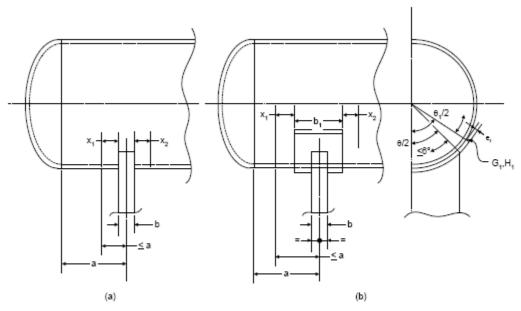


Figure 4.15.2 - Cylindrical Shell Without Stiffening Rings

$$L := L_{ff} \qquad M_{tot} := 12000 kg$$

$$b := 1.5 cm \qquad a := .18 L_{ff} \qquad a = 28.8 \, cm \qquad \theta := 120 deg \qquad \qquad R_m := R_{i_pv} + 0.5 t_{pv}$$

$$b_1 := \min[(b + 1.56 \cdot \sqrt{R_m \cdot t_{pv}}), 2 \cdot a]$$
 $b_1 = 13.04 \text{ cm}$ $b_2 := 20 \text{ cm}$ $k := 0.1$

$$\begin{array}{ll} b_1 \coloneqq \min\Bigl[\bigl(b+1.56\cdot\sqrt{R_m\cdot t_{pv}}\bigr), 2\cdot a\Bigr] & b_1 = 13.04\,\mathrm{cm} \\ \theta_1 \coloneqq \theta + \frac{\theta}{12} & \theta_1 = 130\,\mathrm{deg} \end{array} \qquad \begin{array}{ll} b_2 \coloneqq 20\mathrm{cm} & k \coloneqq 0.1 \\ \end{array}$$

$$Q := 0.5M_{\text{tot}} \cdot g \qquad Q = 5.884 \times 10^4 \text{ N}$$

$$M_{1} := -Q \cdot a \cdot \left[1 - \frac{\frac{a}{L} + \frac{R_{m}^{2} - h_{2}^{2}}{2 \cdot a \cdot L}}{1 - \frac{4h_{2}}{3L}} \right]$$

$$M_{1} := 1.708 \times 10^{3} \text{ N} \cdot \text{m}$$

$$Q \cdot a = 1.695 \times 10^{4} \text{ J}$$

$$M_{2} := \frac{Q \cdot L}{4} \cdot \begin{bmatrix} 1 + \frac{2 \cdot \left(R_{m}^{2} - h_{2}^{2}\right)}{L^{2}} \\ \frac{L^{2}}{1 + \frac{4 \cdot h_{2}}{3L}} - \frac{4a}{L} \end{bmatrix} \qquad M_{2} = 9.971 \times 10^{3} \,\text{N} \cdot \text{m}$$

$$M_{1'} := Q \cdot a$$
 $M_{1'} = 1.695 \times 10^4 \text{ N} \cdot \text{m}$
 $M_{2'} := M_{1'}$ $M_{2'} = 1.695 \times 10^4 \text{ N} \cdot \text{m}$

$$T := \frac{Q \cdot (L - 2a)}{L + \frac{4h_2}{3}}$$

$$T = 3.228 \times 10^4 \,\text{N}$$

4.15.3.3 - long. stresses

distributed load (ASME assumption)

end load (actual)

$$\begin{split} \sigma_1 &\coloneqq \frac{P \cdot R_m}{2 t_{pv}} - \frac{M_2}{\pi R_m^2 t_{pv}} & \sigma_1 = 65.878 \, \text{MPa} & \sigma_{1'} \coloneqq \frac{P \cdot R_m}{2 t_{pv}} - \frac{M_{2'}}{\pi R_m^2 t_{pv}} & \sigma_{1'} = 65.285 \, \text{MPa} \\ \sigma_2 &\coloneqq \frac{P \cdot R_m}{2 t_{pv}} + \frac{M_2}{\pi R_m^2 t_{pv}} & \sigma_2 = 67.574 \, \text{MPa} & \sigma_{2'} \coloneqq \frac{P \cdot R_m}{2 t_{pv}} + \frac{M_{2'}}{\pi R_m^2 t_{pv}} & \sigma_{2'} = 68.167 \, \text{MPa} \end{split}$$

same stress at supports, since these are stiffened, as a<0.5Rm and close to a torispheric head

 $a < 0.5R_{\rm m} = 1$

$$\begin{split} \sigma_{3} &\coloneqq \frac{P \cdot R_{m}}{2t_{pv}} - \frac{M_{1}}{\pi R_{m}^{2} t_{pv}} & \sigma_{3} = 66.58 \, \text{MPa} \\ \sigma_{4} &\coloneqq \frac{P \cdot R_{m}}{2t_{pv}} + \frac{M_{1}}{\pi R_{m}^{2} t_{pv}} & \sigma_{4} = 66.871 \, \text{MPa} \\ \sigma_{4} &\coloneqq \frac{P \cdot R_{m}}{2t_{pv}} + \frac{M_{1}'}{\pi R_{m}^{2} t_{pv}} & \sigma_{4} = 68.167 \, \text{MPa} \end{split}$$

4.15.3.4 - Shear stresses

$$\Delta := \frac{\pi}{6} + \frac{5\theta}{12} \qquad \Delta = 1.396$$

$$\alpha := 0.95 \left(\pi - \frac{\theta}{2}\right) \qquad \alpha = 1.99$$

$$K_2 := \frac{\sin(\alpha)}{\pi - \alpha + \sin(\alpha)\cos(\alpha)} \qquad K_2 = 1.171$$

here we use c), formula for cyl. shell with no stiffening rings and which is not stiffened by a formed head, flat cover or tubesheet. This is worst case, as we havea flange, which can be considered as one half of a stiffening ring pair for each support.

c)
$$\tau_1 := \frac{K_2 \cdot T}{\pi R_m \cdot t_{pv}}$$
 $\tau_1 = 2.198 \,\text{MPa}$ (4.15.14)

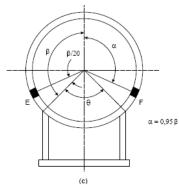
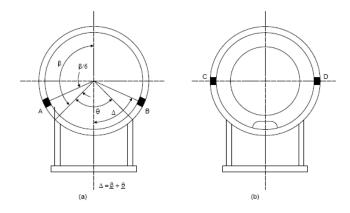


Figure 4.15.5 – Locations of Maximum Longitudinal Normal Stress and Shear Stress in the Cylinder



4.15.3.5 Circumferential Stress

$$K_{5} := \frac{1 + \cos(\alpha)}{\pi - \alpha + \sin(\alpha) \cdot \cos(\alpha)} \qquad K_{5} = 0.76$$

$$\beta := \pi - \frac{\theta}{2} \qquad \beta = 2.094$$

$$K_{6} := \frac{\frac{3 \cdot \cos(\beta)}{4} \cdot \left(\frac{\sin(\beta)}{\beta}\right)^{2} - \frac{5 \cdot \sin(\beta) \cdot \cos(\beta)}{4 \cdot \beta} + \frac{\cos(\beta)^{3}}{2} - \frac{\sin(\beta)}{4 \cdot \beta} + \frac{\cos(\beta)}{4} - \beta \cdot \sin(\beta) \cdot \left[\left(\frac{\sin(\beta)}{\beta}\right)^{2} - \frac{1}{2} - \frac{\sin(2 \cdot \beta)}{4 \cdot \beta}\right]}{2 \cdot \pi \cdot \left[\left(\frac{\sin(\beta)}{\beta}\right)^{2} - \frac{1}{2} - \frac{\sin(2 \cdot \beta)}{4 \cdot \beta}\right]}$$

$$K_{6} = -0.221$$

$$\frac{a}{R_{m}} < 0.5 = 1$$

$$K_{7} := \frac{K_{6}}{4} \qquad K_{7} = -0.055$$

- a) Max circ bending moment
 - 1) Cyl shell without a stiffening ring

$$M_{\beta} := K_7 \cdot Q \cdot R_m$$
 $M_{\beta} = -2.219 \times 10^3 \text{ N} \cdot \text{m}$

c) Circ. stress in shell, without stiffening rings

$$\begin{aligned} x_1 &:= 0.78 \sqrt{R_m \cdot t_{pv}} & x_1 &= 5.77 \text{ cm} & x_2 &:= x_1 \\ \sigma_6 &:= \frac{-K_5 \cdot Q \cdot k}{t_{pv} \cdot \left(b + x_1 + x_2\right)} & \sigma_6 &= -4.288 \, \text{MPa} \end{aligned}$$

$$L < 8R_{\rm m} = 1$$

 $L = 1.6 \,\mathrm{m}$

 $b_1 = 13.04 \, \text{cm}$

$$\sigma_7 := \frac{-Q}{4t_{pv} \cdot \left(b + x_1 + x_2\right)} - \frac{12K_7 \cdot Q \cdot R_m}{L \cdot t_{pv}^2} \qquad \qquad \sigma_7 = 245.974 \text{ MPa}$$
 (4.15.25)

$$\sigma_7 = 245.974 \text{ MPa}$$

too high; we need a reinforcement plate of thickness;

$$t_r := 0.5t_{pv}$$
 strength ratio: $\eta := 1$ (4.15.29)

$$\sigma_{7r} := \frac{-Q}{4(t_{pv} + \eta \cdot t_r) \cdot b_1} - \frac{12K_7 \cdot Q \cdot R_m}{L \cdot (t_{pv} + \eta \cdot t_r)^2} \qquad \sigma_{7r} = 106.188 \text{ MPa}$$
 (4.15.28)

3) f) Acceptance Criteria

$$S := S_{max}$$
 $S = 137.895 \text{ MPa}$

$$|\sigma_{7r}| < 1.25 S_{max} = 1$$

- 4) this section not applicable as $t_r > 2t_{pv} = 0$
- 4.15.3.6 Saddle support, horizontal force given below must be resisted by low point of saddle (where height = h_s)

$$F_{h} := Q \cdot \left(\frac{1 + \cos(\beta) - 0.5 \cdot \sin(\beta)^{2}}{\pi - \beta + \beta \cdot \sin(\beta) \cos(\beta)} \right)$$

$$F_{h} = 5.242 \times 10^{4} \text{ N}$$

$$h_{s} := 9 \text{ cm}$$

$$\sigma_{h} := \frac{F_{h}}{b \cdot h_{s}}$$

$$\sigma_{h} = 38.833 \text{ MPa}$$